

Miloš Adžić, Ana Došen, Kosta Došen, Jovana Kostić, Katarina Maksimović,
Senka Milošević

Is natural deduction natural?

Abstract. Natural deduction, a formalization of hypothetical reasoning, is found in a particular kind of formal systems of logic, which have attracted some attention from cognitive psychologists rather recently. The formal systems envisaged here were not dominant in textbooks of logic. The aim of this work is to answer the question in the title by dealing with such systems of natural deduction, which do not seem to have been envisaged up to now in experimental cognitive psychology. This version of natural deduction is simpler, and stands a chance of being found more natural. We do not present results of experiments, but seek cooperation from psychologists in designing them. The experiments envisaged are to deal with the early growth of logic among children, which are expected to have an at least implicit knowledge of the simple natural deduction rules we would investigate. These are rules concerning first the connectives of conjunction and negation, and also the connectives of implication and disjunction of propositional logic.

Among all the investigations of deductive reasoning in experimental cognitive psychology there are not many that are based on what in logic is called natural deduction. Formal systems based on natural deduction differ from standard formal systems called axiomatic systems of the Hilbert type by giving prominence to rules of inference, and not to axioms, which need not exist. Instead one has something corresponding to what in connection with systems of the Hilbert type is called deduction from hypotheses, and in particular a rule for introducing implication that cancels hypotheses and corresponds to the metatheorem called the Deduction Theorem. The naturalness of this form of deduction would be tied to this hypothetical form, which should have primacy over what used to be called categorical.

Although many logicians, in particular those working in proof theory, believe that this formalization of deduction deserves its attribute “natural”, it cannot be said that this has been confirmed empirically. Natural deduction was not in general central in logic, and still is not, and cognitive psychologists, starting with Jean Piaget (Piaget, 1957; Piaget & Inhelder, 1958), cannot be blamed for not taking account of it in their research of deductive reasoning and human logic. As a matter of fact, psychologists in general did not pay very close attention to deduction as logicians conceive it.

It is only rather recently, by the end of the XX century, that in psychological research, like that of Lance Rips (Rips, 1994), natural deduction made a début. Rips explores the idea--which he calls the Deduction-System Hypothesis--that natural deduction rules are central to cognition because they underlie many other cognitive abilities. For Rips, as for logicians, the hypothetical form of natural deduction has primacy over the categorical and is estimated to be more natural. In his proposal, Rips is interested in the question whether the mental life of human beings embodies certain principles concerning deduction, which are in general a part of the human cognitive architecture and not restricted only to those educated in logic.

In this work the literature concerning this problem is considered and ideas for new experiments are suggested. The authors are not psychologists but logicians, from mathematics and philosophy, and a kindergarten teacher. They wish to attract the attention of psychologists and obtain their help for designing the experiments in a professional manner. The authors offer their competence in logic for obtaining experiments more advanced than they have been up to now.

The accent is on natural deduction as initiated by the logician Gerhard Gentzen (Gentzen, 1969), and not as it is exemplified in most elementary textbooks, in particular the American ones. Natural deduction in the style of Gentzen has deductions in the form of a tree, which records in a better way on which premises a conclusion depends, while in the other tradition derivations are linear and dependencies require additional notes, from which one can obtain instructions for building a tree. Moreover it introduces a hierarchy among the leaves, i.e. hypotheses, of the tree, which is unnecessary both from a mathematical and a practical point of view. This other format was favoured by Rips, presumably because he was more influenced by American textbooks. We favour on the other hand Gentzen's format, and we address the question whether because of its greater simplicity it has psychological advantages too.

Deductions presented by Rips to his experimental subjects, which are mostly students, are rather complex, requiring more than one inference step, and often involving several connectives. We would favour instead simpler deductions with not more than one inference step, and involving just one connective. We do this for two reasons. The first reason is that this simplicity is what one finds in Gentzen's rules of natural deduction, and this, among other things, motivates him and others to call the deductions involved "natural". Moreover, Gentzen's format should be more natural because it does not involve the hierarchy among the hypotheses mentioned in the previous paragraph.

The second reason why we want to deal with Gentzen's simple format is in our projected experimental subjects. We envisage experiments primarily designed for discovering the early growth of logic in the child, namely experiments capable of giving an answer to the question when children learn the meaning of connectives and can base reasoning on this knowledge. We want to see whether in real life this meaning consists in mastering natural deduction rules, as some logicians and philosophers claim. These experiments should investigate how logical reasoning is present in children who understand and use language well, and are not meant to measure the percentage of such children in the whole population.

Although modern logic is closely tied to language, and its doctrines cannot be separated from linguistic matters, as is for example the distinction between syntax and semantics, the design of our experiments should be such so that verbal skills are not decisive. Logic teaches us that in deduction knowledge of language is involved, but this knowledge need not be more than implicit. Our experiments would presuppose not more than this implicit knowledge. Assuming more than that might influence the results in a wrong way. A nonverbal response from an experimental subject should be interpreted as being produced by an implicit deduction as logicians conceive it.

The connectives we envisage are first conjunction, which comes with the word "and", and negation, which comes with the word "not". Knowledge about the meaning of these words is sometimes estimated to be reached already at the age of two (O'Brien, 1998, p. 34). For the connectives of implication, which comes with the word "if", and disjunction, which comes with the word "or", knowledge should be acquired a year later (O'Brien, 1998, p. 34). (It is quite interesting from a logical point of view that these two connectives are here associated.)

Gentzen's rules are rules for introducing a single connective and for eliminating it, and logicians often give precedence to the introduction rules, which sometimes they conceive as a kind of definition. It is not however clear that for every connective in real life the introduction rules are more natural than the elimination rules. One purpose of our experiments would be to try to give a tentative answer concerning this matter too.

Keywords: logic, deduction, natural deduction, reasoning of children, design of experiment

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